# **1. CCRS for Mathematical Practice**

# CCR Standards for Mathematical Practice



### Notes:

Welcome to the College and Career Readiness Standards (CCRS) for Mathematical Practice learning module.

# Introduction



### Notes:

After viewing the module, you will have an understanding of the Standards for Mathematical Practice and how they support students in learning and applying

math. For each math practice, participants will learn what students do when applying the practice and what types of learning activities support the practice. It may be helpful to view the College and Career Readiness Standards (CCRS) Overview - Math module before completing this module if you are new to the CCRS.

# Navigation Tips

Navigating the Module	<b>O</b> O	
<ul> <li>Directions: Click on each of the icons to learn how to navigate the module and access the resources.</li> <li>Image: Click on each of the icons to learn how to navigate the module and access the resources.</li> </ul>		

### Notes:

If this is your first time participating in an online module, please click on each of the icons to learn how to navigate and access resources. When you are ready, please click Next to continue.

# Module Objectives



Notes:

After completing this module, you will be able to state the difference between the CCRS Math Standards and Standards for Mathematical Practice. You will also be able to state the general purpose and benefits of the math practices. You will be able to identify examples of student use for each of the practices. Finally, you will be able to identify learning activities that support student development of the practices, as well as utilize tasks to provide opportunities for your students to use them.

# End of Introduction



### Notes:

Congratulations, you've reached the end of this section. Please advance to the next section to continue the module.

# 2. Introduction

# Introduction to the Standards for Mathematical Practice



### Notes:

Let's begin by discussing the Standards for Mathematical Practice and some benefits of adopting them as part of your math instruction.

The Standards for Mathematical Practice

S	tandards for Mathema	atical Practice	Ø	
	Eight Standards for Mat	hematical Practice		
1.	Make sense of problems and persevere in solving them.			
2.	Reason abstractly and quantitatively.			
з.	Construct viable arguments and critique the reasoning of others.			
4.	Model with mathematics.			
5.	Use appropriate tools strategically.	Click here for the Standards		
6.	Attend to precision.	tool.		
7.	Look for and make use of structure.			
8. Look for and express regularity in repeated reasoning.				

### Notes:

Here are the eight standards for mathematical practice. We will look at each one individually a little later in this module. You may have noticed they do not reference specific math content such as fractions, percent, or addition. Instead, they address

habits people who are proficient at math employ while learning and applying math content.

# Two Sets of Standards



### Notes:

The specific math content adults should learn is addressed in the College and Career Readiness math standards and in the CCRS Overview: Math module. To help avoid confusion around two sets of standards, the College and Career Readiness math standards are typically called the "math content standards" or even just "math standards," while the Standards for Mathematical Practice are commonly referred to as the "math practices."

# Rationale and Benefits of the Math Practices



### Notes:

What is the purpose of the math practices? The main purpose is to define practices that are commonly used by mathematically proficient students. By defining these practices, teachers can modify their instructional approach to help develop the math practices. They can also assign meaningful problems and tasks that provide students the opportunity to use the math practices. Finally, teachers can provide feedback to reinforce the use of math practices when they observe students using them.

# End of Introduction



Congratulations, you've reached the end of this section. Please advance to the next section to continue the module.

# 3. An In-depth Look

# An In-depth Look at Each of the Math Practices



### Notes:

Now, let's take some time to look closely at each of the math practices. There are two handouts that provide extensive information about them as well. The Standards for Mathematical Practice handout provides a narrative summary of each practice. The Math Practice Look-for handout provides a list of things teachers can do to support student use of each practice, along with things they may see students do that demonstrate they are applying each practice. You can download these resources from the resources tab to review at a later time.

An In-depth Look at Each of the Math Practices



### Notes:

The first math practice involves students making sense of problems and persevering in solving them.

# Student Application of Math Practice 1



### Notes:

There are several things students do to apply the first math practice. Obviously, they need to understand the problem. It is often helpful for students to consider different entry points or ways to approach the problem. Consider these options and develop a plan for solving the problem. Applying these strategies helps students make sense of math problems. The perseverance in solving them comes from

monitoring progress and making changes if the initial plan isn't working. It is also important to check answers and consider whether their solution makes sense.



# Teacher Support for Math Practice 1

### Notes:

This slide provides some suggestions for teachers to help students develop the problem-solving strategies needed to apply **Math Practice 1**. First and foremost, allow time for students to understand the problem and to develop a plan to solve it. Asking questions to guide students through the development of their plans and helping students see connections to previous tasks can be especially helpful for students who need to develop their problem-solving skills. Many students do not check their solutions, so taking the time to ask students to discuss and defend their solutions can help them to develop good habits in this regard.

# Problems that Support Math Practice 1



### Notes:

Possibly the most important thing for teachers to do related to Math Practice 1 is to provide appropriate problems for students to work on. A common misapplication teachers have related to this math practice is assuming any word problem students work on is appropriate for this practice. Consider the appropriate level of challenge for your students. They should provide a sufficient challenge to students while not being so complex that they are demoralizing. Typically, problems shouldn't have an obvious approach to solve. Problems that encourage understanding key concepts or that allow multiple strategies to solve are typically appropriate. Ones that allow students to look for a key word to drive their solution strategy are not.

### **Problems that Support Math Practice 1**



**CCR Standards for Mathematical Practice** Last Updated: 02/26/20

### Notes:

A straightforward approach to providing problems that involve conceptual understanding is to provide the typical response to a problem and have students solve the problem, requiring them to figure out how to get there. For example, when going over arithmetic mean, or average, instead of providing data and requiring them to find the average, give them the target average and ask them to figure out how to get there. The first problem listed here is not a bad problem for students trying to master the procedure of finding the mean, but it doesn't really require an understanding of what average represents or any problem solving skills. The second is similar, although it requires students to think about the concept of average and develop a strategy to solve.



# End of an In-depth Look

### Notes:

Congratulations, you've reached the end of this section. Please advance to the next section to continue the module.

# 4. Math Practice 2

# Math Practice 2



### Notes:

The second math practice involves students reasoning abstractly and quantitatively.

# Student Application of Math Practice 2



### Notes:

Math Practice 2 refers to the student's ability to use numbers and words to make sense of quantities. This practice also suggests that students should be able to translate numbers to words and words to numbers. Students should be able to

recognize that quantities can be represented in different ways and consider the relevance of units involved. Finally, students should be able to flexibly use properties of operations.



# Teacher Support for Math Practice 2

### Notes:

Teachers can ask questions that guide students to make sense of quantities. They should also ask students to explain the meanings of symbols they use. Providing opportunities for students to use properties of operations flexibly helps their quantitative reasoning.

# Tasks that Support Math Practice 2



### Notes:

Student work that supports development and use of Math Practice 2 should include tasks that include questions requiring students to recognize the meaning of quantities and their relationships - not just how to compute them. They should also encourage students to convert situations into symbols, requiring students to explain the solution within a meaningful situation. It is also helpful if they contain relevant, realistic content to better enable students to relate to the given quantities.

### **Questions to Develop Mathematical Thinking**



This slide shows some questions teachers can ask students to develop mathematical reasoning.

# End of Math Practice 2



### Notes:

Congratulations, you've reached the end of this section. Please advance to the next section to continue the module.

# 5. Math Practice 3

### Math Practice 3



### Notes:

CCR Standards for Mathematical Practice Last Updated: 02/26/20

The third math practice involves students constructing viable arguments and critiquing the reasoning of others.

# Student Application of Math Practice 3 Math Practice 3 Use properties and definitions to construct arguments. Defend their mathematical reasoning. Listen to arguments of others. Determine validity of others' arguments.

Student Application of Math Practice 3

### Notes:

Math Practice 3 refers to the student's ability to use properties and definitions to construct mathematical arguments and to be prepared to defend their reasoning. This practice also involves listening to and processing the arguments of others to determine their validity.

### **Teacher Support for Math Practice 3**



### Notes:

Teachers can encourage students to use their understanding of math. They can also ask questions encouraging students to compare and contrast approaches to problem-solving strategies. Students should be asked to defend their ideas and arguments. Providing opportunities for students to discuss their arguments and problem-solving strategies is also recommended.

# Tasks that Support Math Practice 3



### Notes:

To support Math Practice 3, student tasks should allow for multiple representations and approaches to promote discussion. This allows students to discuss their reasoning, defend their arguments, and consider the arguments of others.

# Questions to ask to develop mathematical thinking



### Notes:

This slides shows some questions teachers can ask students to develop mathematical thinking. This practice involves students feeling comfortable sharing their thinking with others, which may be difficult at first for some students.

# End of Math Practice 3



### Notes:

Congratulations, you've reached the end of this section. Please advance to the next section to continue the module.

# 6. Math Practice 4

# Math Practice 4



### Notes:

The fourth math practice involves students modeling with mathematics. It is important to note that this is not the same as a teacher modeling how students should work on a math problem. Instead, it involves students developing mathematical models to solve problems in class and the real world.

# **Student Application of Math Practice 4**



### Notes:

Modeling with mathematics is challenging for students, but it can be the ultimate

application of math. It occurs when a student realizes he/she can apply the math he/she has learned to deal with new situations. He/she identifies the important quantities in the given situation, represents it mathematically, and considers whether his/her solution makes sense, given the quantities involved.

# **Teacher Support for Math Practice 4**



### Notes:

Teachers can begin to develop this practice by providing a model for a given situation and asking students whether it is appropriate. They might want to ask students to identify what the solution might look like. Should it be larger than the provided quantities? What units should the answer be given in? They can also ask questions that guide students toward the development of a mathematical model, and ask students to justify it.

# Tasks that Support Math Practice 4



### Notes:

To support Math Practice 4, student tasks should allow students to represent a problem in real-world terms. Students should be able to identify the important quantities and illustrate their relationships using tools such as diagrams, tables, graphs, and formulas. They should analyze these relationships to draw conclusions and interpret whether their results make sense, revising their model if the results seem inappropriate.

### Questions to ask to develop mathematical thinking



### Notes:

This slide shows some questions teachers can ask students to develop

mathematical thinking. This practice involves developing mathematical models, so questions that guide identifying a model and representing given quantities can be helpful.

# End of Math Practice 4



### Notes:

Congratulations, you've reached the end of this section. Please advance to the next section to continue the module.

# 7. Math Practice 5

### Math Practice 5



### Notes:

The fifth math practice involves students using appropriate tools strategically. The ultimate goal is for students to do this naturally. In other words, students independently consider what tools might help when given a situation.

# **Student Application of Math Practice 5**



### Notes:

Math Practice 5 refers to the student's ability to use tools strategically. Obviously, students need to be able to decide or, in many cases, remember to use an appropriate tool to help with a problem. They need to be able to use the selected tool appropriately. Are they using it to make assumptions or predictions, to ensure precision, or to measure something?

# Teacher Support for Math Practice 5



### Notes:

Of course, teachers may need to help students become familiar with some math tools. Providing exposure to a variety of tools, such as calculators, protractors, spreadsheets, and others is a good first step. Students will likely need guidance and practice in using them. Provide opportunities in class for students to decide to use appropriate tools. Don't hand them out or remind students; allow them to make the decision independently. If they don't use a tool that may have helped, ask them after they work on a problem if a specific tool would have helped.

# Questions to ask to reinforce math practice 5



This slides shows some questions teachers can ask students to develop students' use of mathematical tools.

# End of Practice 5



### Notes:

Congratulations, you've reached the end of this section. Please advance to the next section to continue the module.

# 8. Math Practice 6

### Math Practice 6



The sixth math practice involves students attending to precision.

# Student Application of Math Practice 6



### Notes:

**Math Practice 6** involves precision. Obviously, this involves precision in student work, which involves accurate and efficient calculations, along with use of accurate labels and units of measure. It also includes precision in communication, which includes giving the meanings of symbols, using proper math vocabulary, and accurately explaining thinking and solutions while discussing math problems.

### Teacher Support for Math Practice 6



Teachers should emphasize the importance of precision. A misplaced decimal early in a calculation can end up causing a solution to be way off the mark! Strongly encourage efficiency and accuracy in computations and other work. In terms of communication, teachers should model appropriate mathematical language in class and encourage students to do the same. Asking clarification questions can also help students with precision in their communication.

# Tasks and Questions that Support Math Practice 6



### Notes:

In providing students with tasks that relate to precision, consider problems that ask students to round their response, which gets them to consider how precise their answer needs to be. Also, problems that provide different units, such as time presented in hours and minutes, require students to attend to precision in their work.

There are also several questions provided that will help develop precision in your students' work and communication.

# End of Math Practice 6



### Notes:

Congratulations, you've reached the end of this section. Please advance to the next section to continue the module.

# 9. Math Practice 7

### Math Practice 7



### Notes:

The seventh math practice involves students looking for structure and making use of it.

# Student Application of Math Practice 7



### Notes:

Math Practice 7 asks students to identify structures that can help them solve a given problem or address a situation they encounter. It might involve realizing a quantity can be expressed in different ways. An example of this would be a student needing to add 75 and 28. He/she could apply Math Practice 5 and use a calculator. He/she might also recognize that 28 could also be represented as 25 + 3. This allows them to add 75 and 25 to get 100, and then add the remaining 3 to get 103. In this case the student uses the structure of math to simplify his/her calculations to ones he/she is familiar with.

# Student Application of Math Practice 7 (Cont.)



### Notes:

Another way students might make use of structure is to view complicated quantities as single objects or as a composition of several objects. A problem asking students to find the area of an abnormal shape is made easier by looking at the abnormal shape as a combination of more traditional shapes. The shape given here could be looked at as two rectangles and a triangle.

# Teacher Support for Math Practice 7



### Notes:

Teachers can work to illustrate the structure of math while teaching key mathematical concepts. They should also emphasize student understanding of mathematical properties, which often provide a foundation for understanding the structure of math. Students should be engaged in discussion that emphasizes the relationship between various math topics. Teachers should provide activities that allow students to represent mathematics several different ways to help them experience the structure of math.

# Tasks that Support Math Practice 7



### Notes:

To support Math Practice 7, student tasks should obviously require students to find structure in order to solve a problem. Tasks might involve the application of mathematical properties or the manipulation of quantities to better fit the problem. They might ask students to take a complex idea and break it down into component parts that can be used to solve the problem. They could also require students to look at a problem in unconventional ways.

### Questions to ask to develop mathematical thinking



### Notes:

This slides shows some questions teachers can ask students to develop

mathematical thinking.

# End of Math Practice 7



### Notes:

Congratulations, you've reached the end of this section. Please advance to the next section to continue the module.

# **10. Math Practice 8**

### Math Practice 8



The eighth math practice involves students looking for and expressing regularity in repeated reasoning.

# <section-header> Student Application of Math Practice 8 Student Application of Math Practice 8 Math Practice 8 Notice repeated calculations and patterns. Look for general methods or shortcuts. Develop formulas and equations. Substitute fractions, decimals, or large numbers with baseline numbers to clarify a problem.

## Notes:

Students are developing math practice 8 when they notice repeated calculations. They observe patterns in those calculations to find general methods for doing math. For example, a student struggling to add integers might do multiple problems adding numbers with opposite signs and see that the answer is always the difference between the two absolute values with the sign of the larger absolute value. He/she can also use these patterns to develop formulas or equations. For example, a student counting squares to find the area of several rectangles might notice that the area is always the length times the width and derive the formula that way. He/she might initially substitute complex numbers with baseline numbers such as 1, 2, 5, or 10 to clarify a problem and identify a formula or strategy that can be use with more complex numbers.

# **Teacher Support for Math Practice 8**



### Notes:

Teachers should plan to provide a series of problems that are designed to allow students to see the patterns in their work and answers. They may need to ask students if they see patterns in their work or answers. This can also be approached through classroom discussions related to their reasoning. They can emphasize the prerequisite steps sometimes needed to solve problems; this may include scaling the problem down to develop a formula and then applying that formula to the initial problem.

### Tasks that Support Math Practice 8



To support Math Practice 8, student tasks could include problem sets that include repetitive calculations designed to illustrate patterns of solutions. The example provided earlier of adding integers with opposite signs applies here. Tasks involving recognition of patterns to develop mathematical rules or formulas are excellent for this math practice. Also, tasks that allow students to make generalizations are helpful.

# Tasks that Support Math Practice 8 (Cont.)



### Notes:

A good example of this is the "Banquet Problem," which involves square tables that seat one person on each side and asks how many people can sit when various numbers of tables are pushed together. Many students find it helpful to draw pictures as shown here and use the pattern to identify a formula to solve various problems based on this scenario. This type of problem also lends itself well to modeling with mathematics, which is Math Practice 4.

# Questions to ask to develop mathematical thinking



### Notes:

This slides shows some questions teachers can ask students to develop mathematical thinking around finding and using repeated reasoning.

# End of Math Practice 8



### Notes:

Congratulations, you've reached the end of this section. Please advance to the next section to continue the module.

# **11.** Conclusion

## **Objectives Review**



### Notes:

Please take a moment to review the module objectives shown here. How comfortable do you feel with each of the objectives?

### Ending



### Notes:

Congratulations! You have completed this module. Be sure to download any resources you want to save by clicking on the Resources tab. Click on the Exit button below to close the module.